

NAG C Library Function Document

nag_dsycon (f07mgc)

1 Purpose

nag_dsycon (f07mgc) estimates the condition number of a real symmetric indefinite matrix A , where A has been factorized by nag_dsytrf (f07mdc).

2 Specification

```
void nag_dsycon (Nag_OrderType order, Nag_UptoType uplo, Integer n,
                 const double a[], Integer pda, const Integer ipiv[], double anorm,
                 double *rcond, NagError *fail)
```

3 Description

nag_dsycon (f07mgc) estimates the condition number (in the 1-norm) of a real symmetric indefinite matrix A :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since A is symmetric, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

The function should be preceded by a call to nag_dsy_norm (f16rcc) to compute $\|A\|_1$ and a call to nag_dsytrf (f07mdc) to compute the Bunch–Kaufman factorization of A . The function then uses Higham's implementation of Hager's method (see Higham (1988)) to estimate $\|A^{-1}\|_1$.

4 References

Higham N J (1988) FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

1: **order** – Nag_OrderType *Input*

On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order = Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: **order = Nag_RowMajor** or **Nag_ColMajor**.

2: **uplo** – Nag_UptoType *Input*

On entry: indicates how A has been factorized as follows:

if **uplo = Nag_Upper**, $A = PUDU^T P^T$, where U is upper triangular;

if **uplo = Nag_Lower**, $A = PLDL^T P^T$, where L is lower triangular.

Constraint: **uplo = Nag_Upper** or **Nag_Lower**.

3:	n – Integer	<i>Input</i>
<i>On entry:</i> n , the order of the matrix A .		
<i>Constraint:</i> $n \geq 0$.		
4:	a [<i>dim</i>] – const double	<i>Input</i>
Note: the dimension, dim , of the array a must be at least $\max(1, \mathbf{pda} \times n)$.		
<i>On entry:</i> details of the factorization of A , as returned by nag_dsytrf (f07mdc).		
5:	pda – Integer	<i>Input</i>
<i>On entry:</i> the stride separating row or column elements (depending on the value of order) of the matrix in the array a .		
<i>Constraint:</i> $\mathbf{pda} \geq \max(1, n)$.		
6:	ipiv [<i>dim</i>] – const Integer	<i>Input</i>
Note: the dimension, dim , of the array ipiv must be at least $\max(1, n)$.		
<i>On entry:</i> details of the interchanges and the block structure of D , as returned by nag_dsytrf (f07mdc).		
7:	anorm – double	<i>Input</i>
<i>On entry:</i> the 1-norm of the original matrix A , which may be computed by calling nag_dsy_norm (f16rcc). anorm must be computed either before calling nag_dsytrf (f07mdc) or else from a copy of the original matrix A .		
<i>Constraint:</i> $\mathbf{anorm} \geq 0.0$.		
8:	rcond – double *	<i>Output</i>
<i>On exit:</i> an estimate of the reciprocal of the condition number of A . rcond is set to zero if exact singularity is detected or the estimate underflows. If rcond is less than machine precision , A is singular to working precision.		
9:	fail – NagError *	<i>Output</i>
The NAG error parameter (see the Essential Introduction).		

6 Error Indicators and Warnings

NE_INT

On entry, **n** = $\langle value \rangle$.

Constraint: $n \geq 0$.

On entry, **pda** = $\langle value \rangle$.

Constraint: $\mathbf{pda} > 0$.

NE_INT_2

On entry, **pda** = $\langle value \rangle$, **n** = $\langle value \rangle$.

Constraint: $\mathbf{pda} \geq \max(1, n)$.

NE_REAL

On entry, **anorm** = $\langle value \rangle$.

Constraint: $\mathbf{anorm} \geq 0.0$.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to nag_dsycon (f07mgc) involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2n^2$ floating-point operations but takes considerably longer than a call to nag_dsytrs (f07mec) with 1 right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogues of this function are nag_zhecon (f07muc) for Hermitian matrices and nag_zsycon (f07nuc) for symmetric matrices.

9 Example

To estimate the condition number in the 1-norm (or infinity-norm) of the matrix A , where

$$A = \begin{pmatrix} 2.07 & 3.87 & 4.20 & -1.15 \\ 3.87 & -0.21 & 1.87 & 0.63 \\ 4.20 & 1.87 & 1.15 & 2.06 \\ -1.15 & 0.63 & 2.06 & -1.81 \end{pmatrix}.$$

Here A is symmetric indefinite and must first be factorized by nag_dsytrf (f07mdc). The true condition number in the 1-norm is 75.68.

9.1 Program Text

```
/* nag_dsycon (f07mgc) Example Program.
*
* Copyright 2001 Numerical Algorithms Group.
*
* Mark 7, 2001.
*/
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagf16.h>
#include <nagx02.h>

int main(void)
{
    /* Scalars */
    double anorm, rcond;
    Integer i, j, n, pda;
    Integer exit_status=0;
    NagError fail;
    Nag_OrderType order;
```

```

/* Arrays */
char      uplo[2];
Integer *ipiv=0;
double   *a=0;
Nag_UptoType uplo_enum;

#ifndef NAG_COLUMN_MAJOR
#define A(I,J) a[(J-1)*pda + I - 1]
    order = Nag_ColMajor;
#else
#define A(I,J) a[(I-1)*pda + J - 1]
    order = Nag_RowMajor;
#endif

INIT_FAIL(fail);
Vprintf("f07mgc Example Program Results\n\n");

/* Skip heading in data file */
Vscanf("%*[^\n] ");
Vscanf("%ld%*[^\n] ", &n);
#ifndef NAG_COLUMN_MAJOR
    pda = n;
#else
    pda = n;
#endif

/* Allocate memory */
if ( !(ipiv = NAG_ALLOC(n, Integer)) ||
    !(a = NAG_ALLOC(n * n, double)) )
{
    Vprintf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read A from data file */
Vscanf(' %ls %*[^\n] ', uplo);
if (*(unsigned char *)uplo == 'L')
    uplo_enum = Nag_Lower;
else if (*(unsigned char *)uplo == 'U')
    uplo_enum = Nag_Upper;
else
{
    Vprintf("Unrecognised character for Nag_UptoType type\n");
    exit_status = -1;
    goto END;
}

if (uplo_enum == Nag_Upper)
{
    for (i = 1; i <= n; ++i)
    {
        for (j = i; j <= n; ++j)
            Vscanf("%lf", &A(i,j));
    }
    Vscanf("%*[^\n] ");
}
else
{
    for (i = 1; i <= n; ++i)
    {
        for (j = 1; j <= i; ++j)
            Vscanf("%lf", &A(i,j));
    }
    Vscanf("%*[^\n] ");
}
/* Compute norm of A */
f16rcc(order, Nag_OneNorm, uplo_enum, n, a, pda, &anorm, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from f16rcc.\n%s\n", fail.message);
}

```

```

    exit_status = 1;
    goto END;
}
/* Factorize A */
f07mdc(order, uplo_enum, n, a, pda, ipiv, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from f07mdc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
/* Estimate condition number */
f07mgc(order, uplo_enum, n, a, pda, ipiv, anorm, &rcond,
        &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from f07mgc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

if (rcond >= X02AJC)
    Vprintf("Estimate of condition number = %10.2e\n", 1.0/rcond);
else
    Vprintf("A is singular to working precision\n");
END:
if (ipiv) NAG_FREE(ipiv);
if (a) NAG_FREE(a);
return exit_status;
}

```

9.2 Program Data

```

f07mgc Example Program Data
 4                      :Value of N
 'L'                   :Value of UPLO
 2.07
 3.87  -0.21
 4.20  1.87  1.15
 -1.15  0.63  2.06  -1.81  :End of matrix A

```

9.3 Program Results

```

f07mgc Example Program Results
Estimate of condition number = 7.57e+01

```
